

LOGIQUE

Dédution naturelle

1-Axiomes

$$\frac{}{\Gamma, A \vdash A} \text{ax}$$

2-Règles structurelles

$$\frac{\Gamma \vdash A}{\Gamma, B \vdash A} \text{aff}$$

3-Règles des connecteurs

$$\frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A} \wedge^g \text{elim}$$

$$\frac{\Gamma \vdash A \wedge B}{\Gamma \vdash B} \wedge^d \text{elim}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \wedge \text{intro}$$

$$\frac{\Gamma \vdash A \vee B \quad \Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma \vdash C} \vee \text{elim}$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash A \vee B} \vee^g \text{intro}$$

$$\frac{\Gamma \vdash B}{\Gamma \vdash A \vee B} \vee^d \text{intro}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash A \rightarrow B}{\Gamma \vdash B} \rightarrow \text{elim}$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B} \rightarrow \text{intro}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash \neg A}{\Gamma \vdash \perp} \neg \text{elim}$$

$$\frac{\Gamma, A \vdash \perp}{\Gamma \vdash \neg A} \neg \text{intro}$$

$$\frac{\Gamma, \neg A \vdash \perp}{\Gamma \vdash A} \perp \text{classic}$$

4-Règles des quantificateurs

$$\frac{\Gamma \vdash \forall x A}{\Gamma \vdash A[x:=t]} \forall \text{elim}$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash \forall x A} \forall \text{intro} \text{ (si } x \notin \mathcal{VL}(\Gamma) \text{)}$$

$$\frac{\Gamma \vdash \exists x A \quad \Gamma, A \vdash B}{\Gamma \vdash B} \exists \text{elim} \text{ (si } x \notin \mathcal{VL}(\Gamma, B) \text{)}$$

$$\frac{\Gamma \vdash A[x:=t]}{\Gamma \vdash \exists x A} \exists \text{intro}$$

Dédution Naturelle Intuitionniste

1-Axiomes

$$\frac{}{\Gamma, A \vdash A} \text{ax}$$

2-Règles structurelles

$$\frac{\Gamma \vdash A}{\Gamma, B \vdash A} \text{aff}$$

3-Règles des connecteurs

$$\frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A} \wedge^g \text{elim} \qquad \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash B} \wedge^d \text{elim} \qquad \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \wedge \text{intro}$$

$$\frac{\Gamma \vdash A \vee B \quad \Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma \vdash C} \vee \text{elim} \qquad \frac{\Gamma \vdash A}{\Gamma \vdash A \vee B} \vee^g \text{intro} \qquad \frac{\Gamma \vdash B}{\Gamma \vdash A \vee B} \vee^d \text{intro}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash A \rightarrow B}{\Gamma \rightarrow B} \rightarrow \text{elim} \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B} \rightarrow \text{intro}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash \neg A}{\Gamma \vdash \perp} \neg \text{elim} \qquad \frac{\Gamma, A \vdash \perp}{\Gamma \vdash \neg A} \neg \text{intro}$$

$$\frac{\Gamma \vdash \perp}{\Gamma \vdash A} \perp \text{elim}$$

4-Règles des quantificateurs

$$\frac{\Gamma \vdash \forall x A}{\Gamma \vdash A[x:=t]} \forall \text{elim} \qquad \frac{\Gamma \vdash A}{\Gamma \vdash \forall x A} \forall \text{intro} \quad (\text{ si } x \notin \mathcal{V}\mathcal{L}(\Gamma))$$

$$\frac{\Gamma \vdash \exists x A \quad \Gamma, A \vdash B}{\Gamma \vdash B} \exists \text{elim} \quad (\text{ si } x \notin \mathcal{V}\mathcal{L}(\Gamma, B)) \frac{\Gamma \vdash A[x:=t]}{\Gamma \vdash \exists x A} \exists \text{intro}$$