

LOGIQUE

Calcul des séquents

1-Axiomes

$$\frac{}{\perp \vdash \perp} \perp_g \quad \frac{}{A \vdash A} \text{ax}$$

2-Règles structurelles

$$\frac{\Gamma \vdash \Delta}{\Gamma, A \vdash \Delta} \text{aff}_g \quad \frac{\Gamma \vdash \Delta}{\Gamma \vdash A, \Delta} \text{aff}_d$$

$$\frac{\Gamma, A, A \vdash \Delta}{\Gamma, A \vdash \Delta} \text{contr}_g \quad \frac{\Gamma \vdash A, A, \Delta}{\Gamma \vdash A, \Delta} \text{contr}_d$$

3-Règles des connecteurs

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta} \wedge_g \quad \frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta} \wedge_d$$

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta} \vee_g \quad \frac{\Gamma \vdash A, B, \Delta}{\Gamma \vdash A \vee B, \Delta} \vee_d$$

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \rightarrow B \vdash \Delta} \rightarrow_g \quad \frac{\Gamma, A \vdash B, \Delta}{\Gamma \vdash A \rightarrow B, \Delta} \rightarrow_d$$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, \neg A \vdash \Delta} \neg_g \quad \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \neg A, \Delta} \neg_d$$

4-Règles des quantificateurs

$$\frac{\Gamma, A[x:=t] \vdash \Delta}{\Gamma, \forall x A \vdash \Delta} \forall_g \quad \frac{\Gamma \vdash A, \Delta}{\Gamma \vdash \forall x A, \Delta} \forall_d \text{ (si } x \notin \mathcal{VL}(\Gamma, \Delta) \text{)}$$

$$\frac{\Gamma, A \vdash \Delta}{\Gamma, \exists x A \vdash \Delta} \exists_g \text{ (si } x \notin \mathcal{VL}(\Gamma, \Delta) \text{)} \quad \frac{\Gamma \vdash A[x:=t], \Delta}{\Gamma \vdash \exists x A, \Delta} \exists_d$$

5-Règle de coupure

$$\frac{\Gamma \vdash \Delta, A \quad A, \Gamma' \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'} \text{coupure}$$

Calcul des séquents Intuitionniste

1-Axiomes

$$\frac{}{\perp \vdash \perp} \perp_g \quad \frac{}{A \vdash A} \text{ax}$$

2-Règles structurelles

$$\frac{\Gamma \vdash [C]}{\Gamma, A \vdash [C]} \text{aff}_g \quad \frac{\Gamma \vdash \quad}{\Gamma \vdash A} \text{aff}_d$$

$$\frac{\Gamma, A, A \vdash [C]}{\Gamma, A \vdash [C]} \text{contr}_g$$

3-Règles des connecteurs

$$\frac{\Gamma, A, B \vdash [C]}{\Gamma, A \wedge B \vdash [C]} \wedge_g$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \wedge_d$$

$$\frac{\Gamma, A \vdash [C] \quad \Gamma, B \vdash [C]}{\Gamma, A \vee B \vdash [C]} \vee_g$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash A \vee B} \vee_d^1$$

$$\frac{\Gamma \vdash B}{\Gamma \vdash A \vee B} \vee_d^2$$

$$\frac{\Gamma \vdash A, \quad \Gamma, B \vdash [C]}{\Gamma, A \rightarrow B \vdash [C]} \rightarrow_g$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B} \rightarrow_d$$

$$\frac{\Gamma \vdash A}{\Gamma, \neg A \vdash \quad} \neg_g$$

$$\frac{\Gamma, A \vdash \quad}{\Gamma \vdash \neg A} \neg_d$$

4-Règles des quantificateurs

$$\frac{\Gamma, A[x:=t] \vdash [C]}{\Gamma, \forall x A \vdash [C]} \forall_g$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash \forall x A} \forall_d \text{ (si } x \notin \mathcal{VL}(\Gamma) \text{)}$$

$$\frac{\Gamma, A \vdash [C]}{\Gamma, \exists x A \vdash [C]} \exists_g \text{ (si } x \notin \mathcal{VL}(\Gamma, [C]) \text{)}$$

$$\frac{\Gamma \vdash A[x:=t]}{\Gamma \vdash \exists x A} \exists_d$$

5-Règle de coupure

$$\frac{\Gamma \vdash A \quad A, \Gamma' \vdash [C]}{\Gamma, \Gamma' \vdash [C]} \text{coupure}$$